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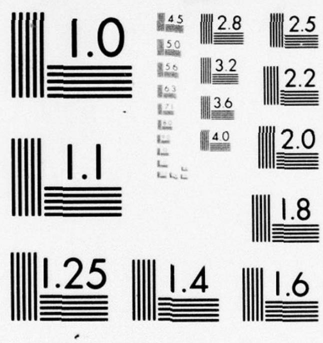

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PERFORMANCE OF THE ADAPTIVE NOISE CANCELLER WITH A NOISY REFERENCE - NON-WIENER SOLUTIONS

M Shensa
(Hydrotronics, Inc)

N00123-78-C-1005

15 March 1979

Final Report: September 1978 — February 1979

Prepared for
Naval Sea Systems Command
Chief of Naval Operations

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The performance of the adaptive noise canceller (ANC) is typically analyzed in terms of Wiener filter theory. However, a narrowband signal in the reference at a frequency for which there is no correlated signal in the primary gives rise to a notch in the output spectrum, a solution which lies outside the scope of the classical theory. In this report, an analytic expression for the output spectrum is derived for the more general case of a reference signal containing a sinusoid in white noise (assumed no reference noise). In general, although the notch depth diminishes with decreasing signal-to-noise ratio, the exact functional relationship depends on the primary input; i.e., there is no longer a time invariant relationship between the output and primary spectra. These results are shown to reduce to those of the classical Wiener filter in the appropriate context.

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I. INTRODUCTION

The performance of the adaptive noise canceller (ANC), pictured in figure 1, has been examined extensively within the framework of Wiener filter theory ([1], [2], [3]). More recently, Glover [4] has shown that for the case of a single sinusoid and no noise in the reference, the output, $\epsilon(k)$, and primary input, $d(k)$, are related by a notched-filter transfer function, a solution which falls outside the scope of the classical theory.

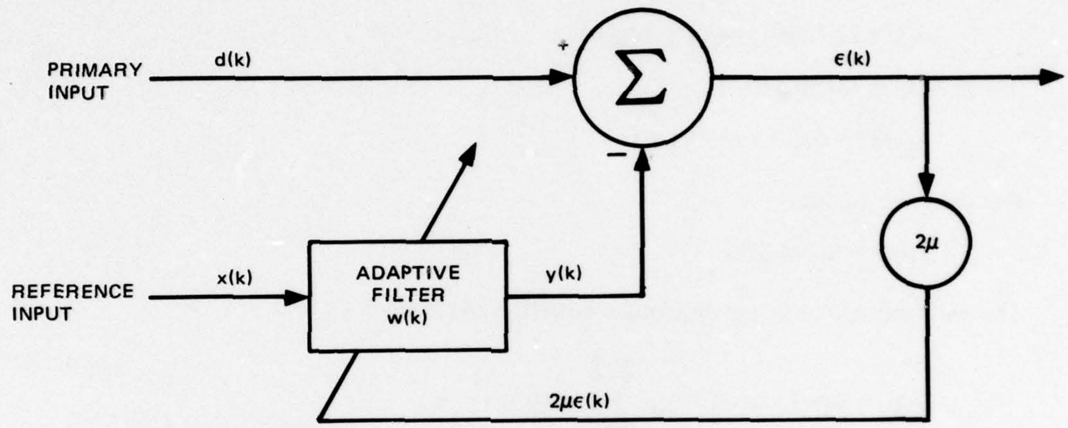

Figure 1. Block diagram of an adaptive noise canceler (ANC). The update equation for the filter weights is $w(k+1) = w(k) + 2\mu\epsilon(k)\,x(k)$.

We examine here a more general situation in which the reference signal consists of a sinusoid (possibly with random phase) in white noise. In that case the output power spectrum no longer bears a time invariant relation to the spectrum of the primary. It is the purpose of this paper to derive an expression for the output spectrum. We obtain an approximate analytic solution which incorporates second order statistical effects (realized as terms of magnitude μ, the feedback parameter). This result is then compared with experimental data and a computer simulation, which prove to be in close agreement with the theory.

It is found that when the primary input is independent of the reference signal, a notch will develop in the output spectrum even though the Wiener solution for the weight vector (the expected value of w; cf [1]) is zero. On the other hand, in the appropriate context (correlated reference and primary), our solution reduces to that of Wiener theory. In general, although the notch depth diminishes with decreasing reference signal-to-noise ratio, the exact functional relationship depends on the nature of the primary input.

II. THEORY

Let the reference input x and adaptive filter w be L-dimensional vectors. We assume that $L \leqslant 2$. The filter is updated using the Widrow LMS algorithm [2],

$$w(k+1) = w(k) + 2\mu\epsilon(k)\,x(k) \quad , \tag{1}$$

where, from figure 1,

$$\epsilon(k) = d(k) - y(k)$$

$$y(k) = w \cdot x = \sum_{\ell=1}^{L} w_{\ell}(k)\, x_{\ell}(k) \tag{2}$$

with subscripts indicating vector components.

The substitution of (2) into (1) yields

$$w(k+1) = A(k)\, w(k) + 2\mu p(k) \quad , \tag{3}$$

where A(k) is the matrix

$$A_{\ell r}(k) = \delta_{\ell r} - 2\mu x_{\ell}(k)\, x_r(k) \tag{4}$$

and p(k) the vector

$$p_{\ell}(k) = x_{\ell}(k)\, d(k) \quad . \tag{5}$$

The solution to the linear difference equation (4) is given by

$$w(k) = B(k,0)\, w(0) + 2\mu \sum_{m=0}^{k-1} B(k,m)\, p(m) \quad , \tag{6}$$

where

$$B(k,m) = \begin{cases} A(k-1)\, A(k-2) \ldots A(m+1) \quad , & m \leqslant k-2 \\ I \quad , & m \geqslant k-1 \end{cases}$$

$$= \prod_{t=m+1}^{k-1} A(t) \quad . \tag{7}$$

The term B(k,0) w(0) is a transient. If the system is stable (as in the case for small μ; cf, equation (B-10) and [5], [6]), $\lim_{k \to \infty} \|B(k)\| = 0$, and for large k the transient is negligible. Alternately, we may assume w(0) = 0. In either case, it follows from (2) and (6) that

$$\epsilon(k) = d(k) - 2\mu \sum_{m=0}^{k-1} x^T(k)\, B(k,m)\, x(m)\, d(m)$$

$$= d(k) - 2\mu \sum_{m=0}^{k-1} f(k,m)\, d(m) \quad , \tag{8}$$

where x^T indicates the transpose of the vector x, and

$$f(k,m) = x^T(k) \left(\prod_{t=m+1}^{k-1} A(t) \right) x(m) \quad . \tag{9}$$

DESCRIPTION OF INPUTS

We assume that the reference input vector has the form

$$x_\ell(k) = C \cos (\omega_o(k - \ell) + \theta) + n_\ell(k) \tag{10}$$

where components of the noise vector, $n_\ell(k)$, are independent random variables with variance σ^2:

$$E[n_\ell(k)\, n_r(k')] = \delta_{\ell r}\, \delta_{kk'}\, \sigma^2 \quad . \tag{11}$$

(E indicates expectation.) Equation (11) is a somewhat stronger condition than in many operational noise cancellers (usually $n_\ell(k) = n(k - \ell)$, a white noise sequence), but is typical of most theoretical treatments. As indicated in Appendix B, it could be circumvented, but is retained for the sake of computational simplicity. As a further computational convenience, we assume that ω_o is bin-centered; i.e.,

$$\omega_o = \frac{2\pi s}{L} \quad , \quad s = \text{integer}. \tag{12}$$

Throughout our development we shall consider x to be a deterministic sinusoid in white noise; i.e., θ is fixed. An alternative viewpoint is that of a narrowband process in white noise modeled by equation (10) with θ a uniform random variable. It should be noted, however, that since our calculations assume θ fixed, all expectations must be interpreted as conditioned on θ. Nonetheless, we shall find that the final results are independent of θ. Consequently, they remain valid from both viewpoints.

Finally, we make the assumption that the primary is a stochastic process independent of the reference; i.e., E(xd) = E(x) E(d) where the expectation E is conditioned on θ in accordance with the remarks above. It is also assumed that the correlation function of the primary is stationary.

From equations (4) and (10)

$$\begin{aligned} S_{\ell r} &= E(A_{\ell r}) \\ &= \delta_{\ell r} - 2\mu[C^2 \cos (\omega_o(k - \ell) + \theta) \cos (\omega_o(k - r) + \theta) + \delta_{\ell r}] \\ &= \delta_{\ell r} - 2\mu \left[\frac{C^2}{2} \cos \omega_o(\ell - r) + \frac{C^2}{2} \cos (\omega_o(2k - \ell - r) + 2\theta) + \delta_{\ell r} \right] \\ &= (I - 2\mu R)_{\ell r} - \mu\, C^2 \cos (\omega_o(2k - \ell - r) + 2\theta) \end{aligned} \tag{13}$$

where

$$R_{\ell r} = \frac{C^2}{2} \cos \omega_o(\ell - r) + \sigma^2 \delta_{\ell r} \tag{14}$$

is the classical correlation matrix of the process x. A necessary condition for stability of the ANC is that the magnitude of the eigenvalues of $I - 2\mu R$ be less than one ([5], [6], Appendix B). In Appendix A, it is shown that $\tilde{S} = I - 2\mu R$ has two eigenvalues λ and $L - 2$ eigenvalues β given by

$$\begin{aligned} \lambda &= 1 - 2\mu \left(C^2 \frac{L}{4} + \sigma^2 \right) \\ \beta &= 1 - 2\mu\sigma^2. \end{aligned} \tag{15}$$

OUTPUT CORRELATION FUNCTION

Ultimately we are interested in the relationship between the spectra of ϵ and d. It would thus be desirable to apply transform techniques. However, it is not clear from equation (8) that the correlation function of ϵ is stationary, let alone the result of time invariant operations (which in general it is not) on d. We therefore proceed carefully.

Denote the correlation function of ϵ by

$$\mathcal{E}(k,k') = E(\epsilon(k)\, \epsilon(k')) \quad . \tag{16}$$

Define

$$G_1(k) = d(k) - 2\mu \sum_{m=0}^{k-1} E[f(k,m)]\, d(m) \tag{17}$$

and

$$G_2(k,k') = 4\mu^2 \sum_{m=0}^{k-1} \sum_{m'=0}^{k'-1} \left(E[f(k,m)\, f(k',m')] - E[f(k,m)\, E[f(k',m')]\right) \; d(m)\, d(m') \quad , \tag{18}$$

where f is defined in equation (9). Since f and d are independent, it follows from (8) and (16) through (18) that

$$\mathcal{E}(k,k') = E[G_1(k)\, G_1(k') + G_2(k,k')] \quad . \tag{19}$$

In succeeding calculations we shall have to make approximations which neglect terms of order μ^2 (denoted $0(\mu^2)$). This does not mean we may ignore G_2 since the summation in equation (18) yields a factor $(1 - \lambda^2)^{-1}$, proportional to μ^{-1}, which with the coefficient μ^2 is of order μ.

It follows from (11) that all the factors of (9) are independent; thus,

$$E[f(k,m)] = \tilde{x}^T(k) \left(\prod_{t=m+1}^{k-1} S(t) \right) \tilde{x}(m) \quad , \tag{20}$$

where

$$\tilde{x}_\ell(k) = C \cos(\omega_o(k-\ell)+\theta) \quad . \tag{21}$$

Furthermore, it is shown in Appendix B that the error in G_1 incurred from replacing $S(t)$ by $I - 2\mu R$ is of order μ^2 (a consequence of the oscillatory behavior with respect to k of the second term in equation (13)). Also equation (A-1) implies that $\tilde{x}$ is an eigenvector of $I - 2\mu R$ with eigenvalue λ. Thus, (20) becomes, approximately,

$$E[f(k,m)] = \tilde{x}^T(k)\, \lambda^{k-m-1}\, \tilde{x}(m). \tag{22}$$

From (21) and (12) we have

$$\tilde{x}^T(k)\,\tilde{x}(m) = C^2 \sum_{\ell=1}^{L} \cos(\omega_o(k-\ell)+\theta)\cos(\omega_o(k-m)+\theta)$$

$$= \frac{C^2 L}{2} \cos \omega_o(\ell - m) \quad . \tag{23}$$

We also note that since $0 < \lambda < 1$, when k is large the sum $\sum_{-\infty}^{0} |Ef(k,m)|$ is arbitraily small.

Consequently, the lower summation limit in (17) may be taken to be $-\infty$ (error $0(\mu^2)$). Finally, equations (22) and (23) imply $E[f(k,m)]$ is a function of $k - m$. Thus, we may rewrite (17) as

$$G_1 = (\delta - 2\mu f_+) * d \quad , \tag{24}$$

where $*$ represents convolution, δ is the discrete Dirac delta function, and

$$f_+(s) = \begin{cases} \dfrac{C^2 L}{2} \lambda^{s-1} \cos \omega_o s \quad , & s \geqslant 0 \\ 0 & s \leqslant 0 \quad . \end{cases} \tag{25}$$

Now let $\tau = k - k'$ and

$$D(\tau) = E[d(k)\, d(k')] \quad . \tag{26}$$

It follows from (24) that

$$E[G_1(k)\, G_1(k')] = (h(s)*h(-s)*D)(\tau) \tag{27}$$

where

$$h(s) = \delta(s) - 2\mu f_+(s) \quad . \tag{28}$$

The expectation of G_2 is calculated in Appendix B (under the assumption $\sum_{\tau=0}^{\infty} |D(\tau)| < \infty$). Combining that result, equation (B-28), with (27) and (19), we have

$$\mathcal{E}(k,k') \doteq \mathcal{E}(\tau)$$

$$= (h(s)*h(-s)*D)(\tau) + 2\mu^2 \sigma^2 C^2 L\, q\, \delta(\tau)$$

$$+ \frac{2\mu^2\sigma^2C^2L}{1-\lambda^2} D(0)\, \lambda|\tau| \cos(\tau\omega_o)$$

$$+ 4\mu^2\sigma^4 \left(\frac{2}{1-\lambda^2} + \frac{L-2}{1-\beta^2}\right) D(0)\, \delta(\tau) \quad , \tag{29}$$

where

$$q = \sum_{s=1}^{\infty} \sum_{s'=1}^{\infty} \lambda^{s-1} \lambda^{s'-1} \cos \omega_o(s-s')\, d(s-s') \quad . \tag{30}$$

Note that the last three terms are of order μ since $\mu^2/(1-\lambda^2)$ is of order μ. It is also clear from (29) that the correlation function of $\epsilon(k)$ depends only on $k - k'$; i.e., it is stationary. Thus, its discrete Fourier transform is proportional to the average of an ensemble of output power spectra (hereinafter simply referred to as the output spectrum).

OUTPUT SPECTRUM

The spectrum of $\mathcal{E}(\tau)$ is given by

$$\hat{\mathcal{E}}(z) = \sum_{\tau=-\infty}^{\infty} \mathcal{E}(\tau) z^{-\tau} \tag{31}$$

where $z = e^{i\omega}$. It follows from the convolution theorem for z-transforms that the transform of $h(s)*h(-s)*D$ is given by $|\hat{h}|^2 \hat{D}$. Also, from (25), we see that

$$\hat{f}(z) = \sum_{s=-\infty}^{\infty} f_+(s)\, z^{-s}$$

$$= \sum_{s=1}^{\infty} \frac{LC^2}{2} \lambda^{s-1} \cos(\omega_o s)\, z^{-s}$$

$$= \frac{LC^2}{2} \frac{\cos \omega_o z - \lambda}{z^2 - 2\lambda z \cos \omega_o + \lambda^2} \quad . \tag{32}$$

Combining this with (28) and $\hat{\delta}(z) \equiv 1$, we have

$$\hat{h}(z) = 1 - 2\mu\,\hat{f}(z) \quad . \tag{33}$$

It is not difficult to show that h(z) has zeroes at

$$z_o = (1 - 2\mu\sigma^2)\, e^{\pm i\omega_o} + 0(\mu^2) \quad . \tag{34}$$

Also, in view of (32), the transform of $\lambda^{|\tau|} \cos \omega_o \tau$ may be written

$$\sum_{s=-\infty}^{\infty} \lambda^{|s|} \cos \omega_o s\, z^{-s} = 1 + \frac{4\lambda}{LC^2}\, \mathrm{Re}\, \hat{f}(z), \tag{35}$$

where Re stands for real part.

Substituting the above results into (29), we have

$$\begin{aligned}\hat{\mathcal{E}}(z) = |\hat{h}(z)|^2\, \hat{D}(z) &+ 2\mu^2\sigma^2\, C^2 L\, q \\ &+ \frac{2\mu^2\sigma^2 C^2 L}{1-\lambda^2}\, D(0) \left(1 + \frac{4\lambda}{LC^2}\, \mathrm{Re}\, \hat{f}(z) \right) \\ &+ 4\mu^2\sigma^4 \left(\frac{2}{1-\lambda^2} + \frac{L-2}{1-\beta^2} \right) D(0) \quad .\end{aligned} \tag{36}$$

It is clear from the last three terms of (36), whose z dependence is independent of $\hat{D}(z)$, that the relation between the output spectrum $\hat{\mathcal{E}}$ and the spectrum D of the primary input is not in general time invariant. Note, however, that in the no-noise case, $\sigma^2 = 0$, these terms vanish, $G_2 \equiv 0$, and the relationship reduces to (24) with transfer function $\hat{h}(z)$ whose zeroes then lie (to order μ^2) on the unit circle. This agrees with the results of Glover in [4] ($2\mu\,\hat{f}(z)$ equals J(z) to order μ^2).

In place of a transfer function, we consider the ratio of the output and input power spectra

$$T(z) = \frac{\hat{\mathcal{E}}(z)}{\hat{D}(z)} \quad , \tag{37}$$

which for $\sigma^2 = 0$ reduces to $|\hat{h}(z)|^2$. We shall examine two cases: d(k) white noise of unit variance, and d(k) a narrowband signal (e.g., sinusoid with random phase) centered at ω_o and of unit amplitude. For white noise, $\hat{D} \equiv 1$, D(0) = 1, and $q = 1/(1 - \lambda^2)$, thus

$$\begin{aligned}T_{noise} = |\hat{h}(z)|^2 &+ \frac{2\mu^2\sigma^2 C^2 L}{1-\lambda^2} + \frac{2\mu^2\sigma^2}{1-\lambda^2} \left(1 + \frac{4\lambda}{LC^2}\, \mathrm{Re}\, \hat{f}(z) \right) \\ &+ 4\mu^2\sigma^4 \left(\frac{2}{1-\lambda^2} + \frac{L-2}{1-\beta^2} \right) \quad .\end{aligned} \tag{38}$$

The autocorrelation function of a sinusoid is given by $D(\tau) = \cos \omega_o \tau$. This function does not satisfy equation (B-1) (also (37) become meaningless); however, we circumvent these difficulties by assuming a slight amount of dampening and taking a finite transform. If we denote the transform length by M, $\hat{D}(e^{\pm i\omega_o}) = M$ and $D(0) = 1$ so that for large M, the first term in (36) will dominate at $z = e^{\pm i\omega_o}$. Thus, although for general ω expression (36) still does not reduce too readily, we have at ω_o

$$\lim_{M \to \infty} T_{sine}(e^{\pm i\omega_o}) = |\hat{h}(e^{\pm i\omega_o})|^2 \ . \tag{39}$$

III. EXPERIMENTAL RESULTS

Both a computer simulation and experiments with a hardware-implemented ANC were performed to evalute the theory of the previous section. The parameters for the computer simulation were $\omega_o = \pi/2$, L = 16, and $\mu = 0.01$. Two hundred twenty-two iterations (i.e., k = 222) were followed by a 128-point discrete Fourier transform (DFT). The input reference power was normalized to one so that the adaptive time constant ([1], [5], [6]) was about 50 iterations. An ensemble of 200 output power spectra were then averaged. The primary input, d(k), and the reference noise, $n_\ell(k)$, consisted of independent pseudo-random white Gaussian noise sequences.

The output spectrum $\hat{\mathcal{E}}(e^{i\omega})$ for a reference signal-to-noise ratio ($SNR = C^2/2\sigma^2$) of 0 dB is found in figure 2. The theoretical spectrum computed from equation (38)

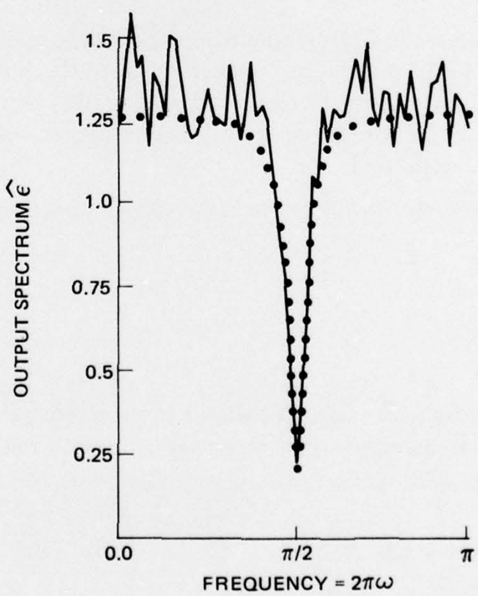

Figure 2. ANC output spectrum for white noise in primary (D = 1) and reference SNR of 0 dB. The filter length is L=16, and μ-.01. The solid line is from a computer simulation; the dotted line is the theoretical spectrum (eq. (38)).

($T(z) = \hat{\mathcal{E}}(z)$ for d white) is also plotted and shows exceptionally good agreement. The same simulation was repeated by replacing the Gaussian noise in reference with noise having a uniform distribution. The differences were negligible. Let us define the notch depth (n.d) as the ratio of the output spectrum to the spectrum of the primary at ω_o:

$$\text{n.d.} = T(e^{\pm i\omega_o}) \quad . \tag{40}$$

A comparison of the theoretical and simulation notch depths for a range of reference SNRs is given in Table 1.

Table 1. Theoretical and simulation notch depths for a reference SNR.

Reference SNR (dB)	6	3	0	-3	-6	-9	-12
Theory	0.072	0.12	0.20	0.32	0.47	0.65	0.82
Simulation	0.11	0.13	0.23	0.34	0.53	0.73	0.87

The theory was also compared with the results of experiments performed on a hardware-implemented ANC with analog inputs [7]. The filter length was $L = 64$, the normalized frequency was $\omega_o = 0.69$ radians and the spectral resolution was 8×10^{-4} radians ($M = 8192$). The reference power varied, producing an equivalent range of μ from 0.00005 to 0.0001. Details may be found in [7]. The reference input noise vector was of the form $n_\ell(k) = n(k - \ell)$. thus violating condition (11). Also, all inputs were low pass filtered at 0.6 Nyquist ($\omega = 1.2\pi$); consequently, the reference noise spectrum was not white.

If we assume the approximations in Appendix B remain valid, the only effects of the colored reference noise spectrum will be on the eigenvalues of R. In particular, a low pass filter with bandwidth BW will result in eigenvalues

$$\lambda = 1 - 2\mu \left(\frac{LC^2}{4} + \frac{\sigma^2 f_s}{2BW} \right)$$

$$\beta = 1 - 2\mu \frac{\sigma^2 f_s}{2BW} \quad , \tag{41}$$

where f_s is the sample rate ([5], [7]). Also, approximately two eigenvalues are λ, $2L\ BW/f_s - 2$ are β, and the remainder are zero. Note that this calls for appropriate changes in the last term of (36).

The experiment was performed for two primary inputs: sinusoidal centered at ω_o and low pass noise. For the noise case, the accuracy of the notch depth measurement accuracy was limited to about 2 dB despite extensive time averaging on a spectrum analyzer. The results, notch depth versus reference SNR, are shown along with the theoretical calculations in figure 3. Excellent agreement was obtained despite the fact that ω_o was not bin centered, the input reference noise vectors were not independent, and theoretical approximations were made to generalize to a low pass rather than white reference noise spectrum.

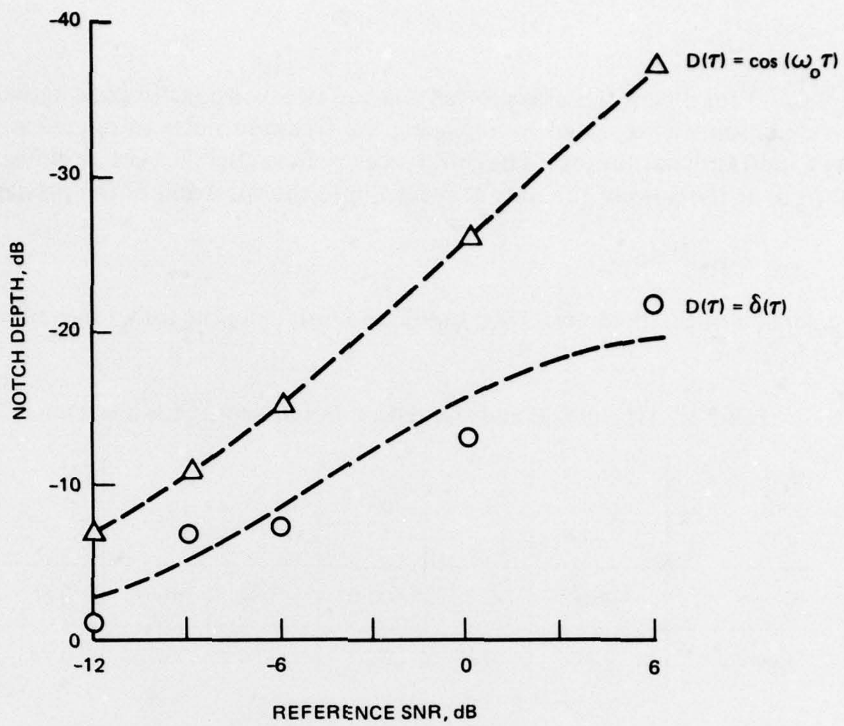

Figure 3. Comparison of notch depths for theory and experiment. The symbol Δ represents experimental results for a sinusoidal primary input, and 0 for a white noise input. The dashed lines are the theoretical curves.

IV. DISCUSSION

It is enlightening to examine our results in the context of classical LMS filter theory; i.e., relate them to the Wiener solution

$$w^* = R^{-1} E(xd) \quad . \tag{42}$$

The basic hypothesis of the Wiener theory – stationary stochastic inputs – may be achieved in the present case by letting the phase θ of equation (10) be random (Section II).

It is well known ([1], [2], [4] through [6]) that the mean of the ANC weight vector, E[w], coverages to w* of equation (42). For a random phase sinusoid in the primary correlated with the reference signal (but not with n(k)), and large M, the power transfer function reduces to (39) (approximately 0 for $\omega \neq \omega_o$) which agrees to order μ^2 with the Wiener solution ([1]). However, when d is independent of x, $w^* = R^{-1} E[xd] = 0$; that is, the expectation of the filter is 0. How then do we explain the presence of a notch filter "transfer function?"

We first note that even when $E(w) = 0$, $E(\|w\|^2) = L\mu\xi_{min}$, the so-called misadjustment noise ([1]). Thus, although the means of the individual weights may be zero, the magnitude of the weight vector remains finite. To confirm these remarks, let us compute

$E(\|w\|^2)$ for two different primary inputs, white noise and sinusoid. Applying the approximations of Appendix B to equation (6), we have

$$E(\|w\|^2) = 4\mu^2 \sum_{m=-\infty}^{k} \sum_{m'=-\infty}^{k} E(p^T(m)\, R^{k-m-1}\, R^{k-m'-1} p(m')) \quad . \tag{43}$$

For d = white noise, $E(p^T(m)\, p(m')) = a\, L\, D(0)\, \delta(m - m')$ where $a = C^2/2 + \sigma^2$; while for $d = \cos(\omega_o k + \theta)$, $E(p^T(m)\, p(m')) = \frac{C^2 L}{4} + b\delta(m - m')$ where b is a constant. It follows from (43) that

$$E(\|w\|^2)_{white} = \frac{a\, L\, D(0) 4\mu^2}{1 - \lambda} = 0(\mu) \tag{44}$$

$$E(\|w\|^2)_{sine} = \frac{LC^2\mu^2}{(1 - \lambda)^2} = 0(1) \quad . \tag{45}$$

The magnitude of (45) is of the order of the misadjustment whereas that of (46) is much larger; i.e., that of the Wiener solution.

Secondly, we note that no matter how small f(k,m) ($\sum_m f(k,m)\, d(m) = w \cdot x$, the filter output) $\hat{f}(z)$ has poles near the unit circle which give rise to a notch in $\hat{h}(z)$. As the noise in the reference increases, the poles move away from the unit circle decreasing the notch depth.

In summary, we have derived (and confirmed by a simulation) an explicit expression, equation (36), for the output spectrum of the adaptive noise canceller when the reference input consists of a sinusoid (narrowband signal) plus white noise. The primary input was assumed independent of the reference noise. When the primary contains a component correlated with the reference sinusoid, the result reduces to the classical Wiener solution. On the other hand, when the primary and reference are independent, the means of the weights are zero. Nevertheless, the narrowband input in the reference gives rise to a second order solution ($E(\|w\|^2) \neq 0$), the weights remain highly correlated with each other, and a notch appears in the output spectrum.

For a noise-free reference, the relationship between the output and primary input is approximately time invariant [4]. In general, however, a time invariant transfer function does not exist, and the output spectrum depends explicitly on the primary, not just its power spectrum. This is reflected in figure 3 which shows a significant difference in notch depth between a white noise primary and a sinusoidal primary.

APPENDIX A: PROPERTIES OF R

Let $G_{\ell r} = \frac{C^2}{2} \cos \omega_o(\ell - r)$. Then $\tilde{x}_\ell(k) = \cos(\omega_o(k - \ell) + \theta)$ is an eigenvector of G.

$$\sum_{r=1}^{L} G_{\ell r} \tilde{x}_r(k) = \sum_{r=1}^{L} \frac{C^2}{2} \cos \omega_o(\ell - r) \cos(\omega_o(k - r) + \theta)$$

$$= \frac{LC^2}{4} \cos(\omega_o(k - \ell) + \theta)$$

$$= \frac{LC^2}{4} \tilde{x}_\ell(k) \quad , \tag{A-1}$$

since $\omega_o = \frac{2\pi s}{2}$ implies $\sum_{r=1}^{L} \cos(\omega_o r + \phi) = 0$.

It is easy to see that $\sin(\omega_o(k - \ell) + \theta)$ is an eigenvector of G with the same eigenvalue. Furthermore $G_{\ell r} = \frac{C^2}{2} (\cos \omega_o \ell \cos \omega_o r - \sin \omega_o \ell \sin \omega_o r)$ is of rank ≤ 2. It follows that G has two eigenvalues equal to $\frac{LC^2}{4}$ and the remainder zero ([5], [6]). The matrix R equals $G + \sigma^2 I$. Let $\tilde{S} = I - 2\mu R$. Then $\tilde{S}$ has two eigenvalues λ and $L - 2$ of β where

$$\lambda = 1 - 2\mu \left(\frac{C^2 L}{4} + \sigma^2 \right)$$

$$\beta = 1 - 2\mu\sigma^2 \quad . \tag{A-2}$$

Also, the vector $\tilde{x}$ is an eigenvector of $\tilde{S}$ with eigenvalue λ, and the trace of $\tilde{S}^k$ is

$$tr(\tilde{S}^k) = 2\lambda^k + (L - 2)\beta^k \quad . \tag{A-3}$$

APPENDIX B: APPROXIMATION OF $E[G_2(k,k')]$

The central idea is to approximate G_2 by replacing the matrix factors A(t) in (18) by their expected values. Note from (9) and (18) that there exist pairs of factors which are correlated. We shall assume that the correlation function of the primary, $D(\tau)$, is absolutely integrable

$$\sum_{t=0}^{\infty} |D(\tau)| = a_o < \infty \quad . \tag{B-1}$$

Let us write (18) as

$$E[G_2(k,k')] = \sum_{m=0}^{k-1} \sum_{m=0}^{k'-1} g(m,m') \tag{B-2}$$

and let $\tilde{g}$ be obtained from g by replacing A with S = E(A). For small μ, we shall demonstrate the existence of n_o such that

$$\sum_{m=k-n_o}^{k-1} \sum_{m'=k'-n_o}^{k'-1} |g(m,m') - \tilde{g}(m,m')| < e_1 \tag{B-3}$$

and

$$\left| \sum_{m=-\infty}^{k-n_o-1} \sum_{m'=-\infty}^{k'-n_o-1} g(m,m') \right| < e_2 \, , \quad \left| \sum_{m=-\infty}^{k-n_o-1} \sum_{m'=-\infty}^{k'-n_o-1} \tilde{g}(m,m') \right| < e_2 \, , \tag{B-4}$$

where terms of magnitude e_1 and e_2 have a negligible effect on $E(G_2)$. It then follows that for $k, k' > n_o$

$$\sum_{m=0}^{k} \sum_{m\ 0}^{k'} g(m,m') \sim \sum_{m=k-n_o}^{k} \sum_{m'=k'-n_o}^{k'} g(m,m')$$

$$\sim \sum_{m=-\infty}^{k} \sum_{m'=-\infty}^{k'} \tilde{g}(m,m') \quad , \tag{B-5}$$

where the error in approximation is of the order $e_1 + 2e_2$.

SUBSTITUTION OF S = E(A) FOR A

Replacing A by E(A) in the second term of g(equations (B-2) and (18)) produces no error since the factors comprising f(k,m) are uncorrelated. Thus we confine our attention to the first term

$$E[f(k,m)\, f(k',m')] = E[x^T(k) \prod_{t=m+1}^{k-1} A(t)\, x(m)\, x^T(k') \prod_{t'=m'+1}^{k'-1} A(t')\, x(m')] . \quad \text{(B-6)}$$

The matrix A(t) is correlated with A(t′) only for t and t′ ranging from 1 + max(m,m′) to min(k,k′) -1. Denote these limites by t_o and t_1, respectively. The corresponding factors (possibly the empty set) give rise to terms containing the factor

$$E\left[\sum_{r_j, r_j'} A_{r_1 r_2}(t_o)\, A_{r_2 r_3}(t_o+1) \ldots A_{r_k r_\ell}(t_1)\, A_{r_1' r_2'}(t_o) \ldots A_{r_k' r_\ell'}(t_1)\right] \quad \text{(B-7)}$$

which is independent of the remainder of (B-6).

Let $Q = A \otimes A$ be the tensor product of A with itself,

$$Q_{r_1 r_1'; r_2 r_2'} = A_{r_1 r_2} A_{r'_1 r'_2} \quad . \quad \text{(B-8)}$$

It then follows from (4), (10), and (B-8) that

$$E(Q) = S \otimes S + V \quad , \quad \text{(B-9)}$$

where V is a non-negative matrix satisfying $\|V\| \leqslant a_1 \mu^2$ for some constant $a_1 \geqslant 0$ (provided the fourth moment of the noise is bounded). Let $T = S \otimes S$ and λ_T indicate the maximum eigenvalue of T. Then

$$\lambda_T = 1 - a_2\mu + 0(\mu^2)$$

$$\lambda_V \leqslant a_1 \mu^2$$

$$\lambda_Q = 1 - a_3\mu + 0(\mu^2), \quad \text{(B-10)}$$

where $0 \leqslant a_1$ and $0 \leqslant a_3 \leqslant a_2$. We shall assume that μ is small enough so that $0 < \lambda_T, \lambda_V, \lambda_Q < 1$. Since $Q^n = (T+V)^n - T^n$, we have

$$\|Q^n - T^n\| \leqslant \sum_{j=1}^{n} \binom{n}{j} \|V\|^j \|T\|^{n-j}$$

$$\leqslant (\lambda_T + \lambda_V)^n - \lambda_T^n$$

$$\leqslant \left(1 + \frac{\lambda_V}{\lambda_T}\right)^n - 1 \quad . \quad \text{(B-11)}$$

Secondly, we note that the vectors x are correlated with A(t) in (B-6) only if $k \neq k'$ and/or $m \neq m'$. In that case a typical factor is

$$E(x_s(k)\, A_{qr}(t')) = E(x_s)\, E(A_{qr}) + a_4\mu + 0(\mu^2) \quad . \tag{B-12}$$

There are at most two such factors in (B-6).

Let $n_1 = k - m$, $n_2 = k' - m'$, and $n_3 = t_1 - t_0 + 1$. We first replace Q by T in (B-6) and then make the substitution (B-12). Assuming the first two moments of the vector x to be bounded and taking into account equations (18), (B-2), (B-6), and (B-8) through (B-12), we have

$$\sum_{m=k-n_o}^{k-1} \; \sum_{m'=k'-n_o}^{k'-1} |g(m,m') - \tilde{g}(m,m')|$$

$$\leq \mu^2 a_5 \sum_{n_1=0}^{n_o-1} \; \sum_{n_2=0}^{n_o-1} \left(z_{n_3}\, \lambda_S^{\,n_1+n_2-2n_3} + a_4\mu\, \lambda_S^{\,n_1+n_o} \right)$$

$$\cdot\, |D(n_1 - n_2 \pm k - k')| \quad , \tag{B-13}$$

where a_5 is a constant, λ_s is the maximum eigenvalue of S, and

$$z_n = \left(1 + \frac{\lambda_V}{\lambda_T}\right)^n - 1 \quad . \tag{B-14}$$

Note that $\lambda_s < 1$, z_n is an increasing function of n, and $n_3 \leq \min(n_1, n_2)$. The above sum is bounded by twice the sum over $n_1 \geq n_2$, thus if we let $\tau = n_1 - n_2$, inequality (B-13) implies

$$\sum_m \sum_{m'} |g(m,m') - \tilde{g}(m,m')|$$

$$\leq \mu^2 a_5 (z_{n_o} + a_4)\, 2 \sum_{n_1=0}^{n_o} \; \sum_{\tau=0}^{n_1} \lambda_s^{\tau} |D(\tau \pm k - k')|$$

$$\leq \mu^2 a_5 (z_{n_o} + a_4\mu)\, 2\, n_o a_o \quad , \tag{B-15}$$

where we have used (B-1).

Similarly,

$$\sum_{m=-\infty}^{k-n_o-1} \sum_{m'=-\infty}^{k'-n_o-1} |g(m,m')|$$

$$\leqslant a_6\mu^2 \sum_{n_1=n_o}^{\infty} \sum_{n_2=n_o}^{\infty} \lambda_Q^{\left(\frac{n_1+n_2}{2}\right)} |D(n_1 - n_2 \pm k - k')|$$

$$\leqslant \frac{2a_6\mu^2}{1-\lambda_Q} a_o \lambda_Q^{n_o} \quad . \qquad \text{(B-16)}$$

Since $\lambda_Q \geqslant \lambda_T$, the same approximation holds for $\widetilde{g}$.

It is straightforward to show that $\lim_{n\mu \to \infty} (1 - a_3\mu)^n = 0$ uniformly in μ in a neighborhood of zero. It thus follows from (B-10) that given $c_1 \geqslant 0$ arbitrarily small, there exists c_2 such that

$$\mu n \geqslant c_2 \Rightarrow \lambda_Q^n \leqslant c_1 << 1 \quad . \qquad \text{(B-17)}$$

Relation (B-17) is roughly equivalent to the statement that the adaptive time constant of the ANC is inversely proportional to μ ([1], [5], [6]).

Also, it follows from (B-10) that $\lambda_V/\lambda_T = a_1\mu^2 + 0(\mu^3)$. If we set $b = \mu n$,

$$z_{b/\mu} = (1 + a_1\mu^2 + 0(\mu^3))^{b/\mu} - 1 \quad . \qquad \text{(B-18)}$$

Taking logarithms and applying L' Hospital's rule, we find that $\lim_{\mu \to 0} z_{b/\mu} = 0$ (in fact $z_{b/\mu} = 0(\mu)$). Thus, given c_3, we can find μ_o' dependent only on b such that $\mu < \mu_o'$ implies $bz_{b/\mu} < c_3$. We then pick $\mu_o < \mu_o'$ such that

$$\mu < \mu_o \Rightarrow (z_{b/\mu} + a_4\mu)\, b < 2c_3 \quad . \qquad \text{(B-19)}$$

Now, let $b = c_2$, pick μ_o such that (B-19) holds, and given any $\mu < \mu_o$, choose $n_o = b/\mu$. Then (B-17) also holds. The inequalities (B-15) through (B-17) and (B-19) then imply (B-3) and (B-4) with

$$e_1 = \mu\, 4a_oa_5c_3$$

$$e_2 = \frac{\mu^2}{1-\lambda_Q} 2a_oa_6c_1 \qquad c_1, c_3 << 1 \quad . \qquad \text{(B-20)}$$

(Briefly, $c_1 \Rightarrow c_2$; $c_2, c_3 \Rightarrow \mu_o$; $\mu, c_2 \Rightarrow n_o$; c_1 and c_3 are arbitrary, a_5 and a_6 depend on x, a_o depends on D.)

APPROXIMATION OF S BY $\tilde{S}$ = I - 2μR

It remains to evaluate $\tilde{g}(m,m')$:

$$\tilde{g}(m,m') = 4\mu^2 \left(E\left[x^T(k) \prod_{t=m}^{k-1} S(t)\, x(m) x^T(k') \prod_{t'=m'}^{k'-1} S(t')\, x(m') \right] \right.$$

$$\left. - E\left[x^T(k) \prod_{t=m}^{k-1} S(t)\, x(m) \right] E\left[x^T(k') \prod_{t'=m'}^{k'-1} S(t')\, x(m') \right] \right) D(m - m') \;. \quad \text{(B-21)}$$

It follows from equations (12) and (13) that

$$\left(\prod_{t=t_o}^{t_o+L} S(t) \right)_{\ell r} = \left(\prod_{t=t_o}^{t_o+L} (I - 2\mu R) \right)_{\ell r} - \sum_{t=t_o}^{t_o+L} \mu C^2 \cos(\omega_o(2t - \ell - r) + \theta) + 0(\mu^2)$$

$$= \left(\prod_{t=t_o}^{t_o+L} (I - 2\mu R) \right)_{\ell r} + 0(\mu^2) \;. \quad \text{(B-22)}$$

Then,

$$\prod_{t=m}^{k-1} S(t) = ((I - 2\mu R)^{k-m} + 0(\mu^2)^{\frac{k-m-1}{L}} + a_7\mu, \quad \text{(B-23)}$$

where the constant, a_7, arises from a product of at most L – 1 terms (i.e., accounts for the case (k – 1 – m)/L not integral) and is independent of k and m. Expressions (B-11) and (B-23) imply that the error incurred in replacing S by I – 2μR in the above product is bounded by

$$(1 + a_8\,\mu^2)^n - 1 + a_7\mu \quad ; \quad n = (k - m - 1)/L \;. \quad \text{(B-24)}$$

Repeating our previous arguments (cf (B-19)), we see that (B-21) remains valid under the additional substitution of $\tilde{S}$ = I – 2μR for S.

A similar application of (B-24) justifies the same substitution in equation (20). Note that in that case although the coefficient of the term is only μ (as compared with μ^2 in (B-15)) there is only a single summation over m. Hence the factor n_o is absent and the error is still of order $0(\mu^2)$. These same techniques could be used to relax condition (11) to $n_\ell(k) = n(k - \ell)$ where n is white; however, we refrain from doing so in order to avoid complicating what are already fairly unwieldy calculations.

COMPUTATION OF $E(G_2)$

Since in (B-2) $k > m$ and $k' > m'$, we have from (10) and (21)

$$E[x_r(k)\, x_{r'}(k')\, x_\ell(m)\, x_{\ell'}(m')] = \tilde{x}_r(k)\, \tilde{x}_{r'}(k')\, \tilde{x}_\ell(m)\, \tilde{x}_{\ell'}(m')$$

$$+ \sigma^2 \delta(k-k')\, \delta(r-r')\, \tilde{x}_\ell(m)\, \tilde{x}_{\ell'}(m')$$

$$+ \sigma^2 \delta(m-m')\, \delta(\ell-\ell')\, \tilde{x}_r(k)\, \tilde{x}_{r'}(k')$$

$$+ \sigma^2 \delta(k'-m)\, \delta(r'-\ell)\, \tilde{x}_r(k)\, \tilde{x}_{\ell'}(m')$$

$$+ \sigma^2 \delta(k-m')\, \delta(r-\ell')\, \tilde{x}_{r'}(k')\, \tilde{x}_\ell(m)$$

$$+ \sigma^4 \delta(k-k')\, \delta(r-r')\, \delta(m-m')\, \delta(\ell-\ell') \quad . \tag{B-25}$$

These six terms appear in the expansion of the first term of (B-21). The evaluation of (B-21) is now straightforward. We note that the occurrence of the first term of (B-25) in (B-21) cancels the entire second term of (B-21).

For illustration, we carry through the substitution of the second term of (B-25) into (B-21). It follows from Appendix A that $\tilde{x}$ is an eigenvector of $\tilde{S}$ with eigenvalue λ. Replacing S by $\tilde{S}$ in (B-21) as justified above, and substituting the second term of (B-25), we have

$$4\mu^2 \sum_{r=1}^{L} \sum_{r'=1}^{L} \sigma^2 \delta(k-k')\, \delta(r-r')\, \lambda^{k-m-1}\, \tilde{x}_r(m)\, \lambda^{k'-m-1}\, \tilde{x}_{r'}(m')\, D(m-m')$$

$$= 4\mu^2\sigma^2 C^2\, \delta(k-k')\, \lambda^{k-m-1}\, \lambda^{k'-m'-1}\, D(m-m') \sum_{r=1}^{L} \cos(\omega_o(m-r)+\theta)$$

$$\cos(\omega_o(m'-r)+\theta)$$

$$= 4\mu^2\sigma^2 C^2\, \frac{L}{2}\, \delta(k-k')\, \lambda^{k-m-1}\, \lambda^{k'-m'-1} \cos\omega_o(m-m')\, D(m-m') \quad . \tag{B-26}$$

The contribution of (B-26) to $E[G_2]$ may be calculated by summing over m and m'

$$\sum_{m=-\infty}^{k} \sum_{m'=-\infty}^{k} 2\mu^2\sigma^2 C^2 L\, \delta(k-k')\, \lambda^{k-m-1}\, \lambda^{k'-m'-1} \cos\omega_o(m-m')\, D(m-m')$$

$$= 2\mu^2\sigma^2 C^2 L\, \delta(k-k') \sum_{s=1}^{\infty} \sum_{s'=1}^{\infty} \lambda^s \lambda^{s'} \cos\omega_o(s-s')\, D(s-s') \quad . \tag{B-27}$$

A reasonable amount of similar algebra including the application of (A-2) and (A-3) yields

$$E[G_2(k,k')] = F_1 + F_2 + F_3 + F_4 \quad , \tag{B-28}$$

where

$$F_1 = 2\mu^2\sigma^2 \, C^2 \, L \, q \, \delta(k - k')$$

$$F_2 = 2\mu^2\sigma^2 \, C^2 \, L \, \frac{D(0)}{1 - \lambda^2} \, \lambda^{|k-k'|} \cos \omega_o(k - k')$$

$$F_3 = 4\mu^2\sigma^4 \left(\frac{2}{1 - \lambda^2} + \frac{L - 2}{1 - \beta^2} \right) D(0) \, \delta(k - k')$$

$$F_4 = 2\mu^2\sigma^2 C^2 L \, (q_c \cos \omega_o(k - k') - q_s \sin \omega_o \, |k - k'|) \, \lambda^{|k-k'-1|} \, (1 - \delta(k - k'))$$

and

$$q = \sum_{s=1}^{\infty} \sum_{s'=1}^{\infty} \lambda^{s-1} \lambda^{s'-1} \cos \omega_o \, (s - s') \, D(s - s') \, D(s - s')$$

$$q_c = \sum_{s=1}^{\infty} \lambda^{s-1} \cos \omega_o s \, D(s) \quad ; \quad q_s = \sum_{s=1}^{\infty} \lambda^{s-1} \sin \omega_o s \, D(s) \quad .$$

The bracketed expression in F_3 containing the eigenvalues λ and β of $\widetilde{S}$ arises from summing the trace of $\widetilde{S}^{2k-2m-2}$ over m. F_4 results from combining the effects of term four and five of (B-25).

Since λ and β are of the form $1 - m\mu + 0(\mu^2)$, the quantities $\mu^2/(1 - \lambda^2)$ and $\mu^2/(1 - \beta^2)$ are of the order μ. Making use of (B-1), we can also show that $q \leqslant a_o/(1 - \lambda^2)$ and that q_c and q_s are uniformly bounded in μ. Thus F_1, F_2, and F_3 are of order μ, whereas F_4 is of order μ^2.

We also note that $\mu^2/(1 - \lambda_Q)$ in (B-20) is of order μ. Since c_1 and c_3 may be taken arbitrarily small as μ goes to 0, we have $\lim_{\mu \to 0} e_1/\mu = 0$ and $\lim_{\mu \to 0} e_2/\mu = 0$, This implies that e_1 and e_2 are negligible compared with terms of order μ for small μ. Thus, our approximations are correct to order μ, and we may also neglect F_4 in equation (B-28).

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